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Survey on Practical Applications of Fuzzy Rule Interpolation

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Abstract

In the last thirty years fuzzy logic became very popular. One can find solutions based on it in several fields from industrial systems to house appliances. Recently a new category of fuzzy systems gained more attention, the so called fuzzy rule interpolation (FRI) based systems. Owing to the low complexity of their rule bases, i.e. they can infer as well when only the relevant rules are known, they can be applied successfully even in cases when a traditional fuzzy system could not give an interpretable result in lack of the full coverage of the rule base.

In this paper, after doing a survey on FRI methods we present several successful practical applications organized in three main areas, namely fuzzy control, function approximation and expert systems.

Keywords: fuzzy rule interpolation; sparse rule bases; application examples; function approximation; rule based control

1. Introduction

Traditional fuzzy reasoning methods (e.g. the Zadeh-Mamdani type compositional rule of inference) demand complete (covering) rule bases, and therefore the construction of a corresponding rule base requires a special care in order to create all the possible rules. Covering rule bases are characterized by a high number of rules that grow exponentially with the number of antecedent dimensions and the number of linguistic terms. In order to solve the complexity problem sparse (not covering) rule bases and inference methods based on rule interpolation can be applied [1]. A fuzzy rule base is sparse if for one or more possible input values it does not contain any matching rules.

A sparse fuzzy rule base can arise in two ways. The first one starts from a completely covering rule base and reduces the number of the rules excluding the non relevant rules or merging the similar ones. The methods following the second way produce a sparse rule base directly. Usually they apply one of the following approaches:

1. Try to identify the so-called optimal fuzzy rules (e.g. [2]).
2. Extend the rule base by applying the concept of Rule Base Extension (e.g. [3]).
3. Create the starting rules based on fuzzy clustering [4][5][6]).
4. Apply evolutionary algorithms (e.g. [7][8]).

Having a sparse rule base one may use an approximate inference technique for fuzzy reasoning. In most of the cases the procedure developed for this purpose determines the conclusion using a fuzzy rule interpolation (FRI) method. FRI methods can be divided into two groups depending on whether

1. they produce the conclusion directly (one-step FRI methods) or,
2. first they interpolate an auxiliary rule and then they calculate the consequent by using that rule (two-step FRI methods).

Traditional fuzzy logic based systems have been successfully used in fields of control (e.g. [9][10][11]), fuzzy modeling (e.g. [12]) and expert systems (e.g. [13]). FRI methods became popular since the second half of the 1990s owing to their applicability in cases with reduced amount of information (sparse rule base). Several successful applications have been reported in the literature in different fields. In this paper, we give a survey on some of them emphasizing the typical application areas.

The rest of the paper is organized as follows. Section 2 gives a short introduction on the FRI techniques. Section 3 presents the practical applications organizing them in three main groups.

2. Survey on Fuzzy Rule Interpolation Methods

2.1. One-step Fuzzy Rule Interpolation Methods

The techniques belonging to the first group of FRI methods produce the conclusion directly based on the observation and two or more neighboring rules. The base method of this art is the KH interpolation, which initiated the FRI research. There are numerous descendants, which overcame the delimitations of the first linear rule interpolation technique and improved as well as completed the base method.

The key idea of the **linear rule interpolation** proposed by Kóczy and Hirota (KH method) [1] is that the approximated conclusion divides the distance between the consequents of the two nearest rules in the same ratio as the observation divides the distance between the antecedent sets of the same rules. This solution is called Fundamental Equation of the fuzzy Rule Interpolation (FERI). The method is α-cut based, and the above mentioned ratio is calculated for each cut separately for the lower and upper distances. The applied function is (1):

$$d(A^*, A_1) : d(A^*, A_2) = d(B^*, B_1) : d(B^*, B_2) . \qquad (1)$$

The KH method was developed originally for Single Input Single Output (SISO) fuzzy systems, but it was extended for the case of Multiple Input Single Output (MISO) fuzzy systems as well by using Minkowski type distances.

The **Modified α-Cut based Interpolation** (MACI) was published by Tikk and Baranyi [14]. It applies a vector representation of the fuzzy sets and transforms the calculations into a vector space where the possibility of the abnormal consequent sets is eliminated.

The MACI method describes every fuzzy set by the help of two vectors, which contain the abscissa values of the left (bottom) and right (upper) flanks of the set. In case of smooth membership functions the endpoints of the α-cuts form the vectors.

An advantage of the method is that always results valid fuzzy sets, and it can be extended for MISO systems as well. Besides, the generalized version of MACI [15] can handle non-convex fuzzy sets, too. As a drawback one can mention that MACI does not preserve the piecewise linearity. However, the deviation is smaller than in the case of the KH method [14].

The **Fuzzy Interpolation in the Vague Environment** (FIVE) developed by Kovács and Kóczy [16][17] and extended by Kovács [18] applies a new approach by placing the task of fuzzy rule interpolation into a virtual space, the so-called vague environment. The concept of the vague environment is based on the similarity and indistinguishability of the objects. In the vague environment the similarity of two fuzzy sets is described by their weighted distance, where the so-called scale function is the weighting factor. The scale function describes the shapes of the fuzzy sets of a fuzzy partition. The method does not preserve the piecewise linearity.

The application of the method is restricted by the need on an exact or approximate universal scale function for each dimension, which describes the whole partition even if the partition is not of Ruspini type. After defining the vague environment of the antecedent and consequent universes each rule will be represented by a point in the vague environment of the rule base and the position of the conclusion can be calculated by linear interpolation.

The vague environments for the antecedent and the consequent sides can be produced beforehand. This speeds up the method, because in course of the inference only the interpolation needs to be done.

The method is applicable in MISO cases as well. FIVE is application-oriented, because it is fast and easy, thus it can be embedded into direct robot control, too.

2.2. Two-step Fuzzy Rule Interpolation Methods

The two-step fuzzy rule interpolation methods follow the concept of the **Generalized Methodology of fuzzy rule interpolation** (GM) suggested by Baranyi, Kóczy and Gedeon (e.g. in [19]). GM characterizes the position of fuzzy sets by reference points. In its first step it interpolates a new rule in the same location as the position of the observation. Thus the reference point of each antecedent linguistic term of the new rule is identical the reference point of the observation set in the corresponding dimension. The first step consists of three sub-steps:

- Determine the antecedent set shapes of the interpolated rule by the help of a set interpolation.

- Determine the location of the consequent sets by the help of a crisp interpolation/extrapolation method.
- Determine the consequent set shapes of the interpolated rule using the same set interpolation technique as in the case of the antecedent sets.

The conclusion corresponding to the observation is produced in the second step by the help of this rule. As the antecedent part of the estimated rule generally does not fit perfectly the observation, therefore some kind of special single rule reasoning is needed. Several techniques are suggested in [19] for this task (e.g. FPL, SRM-I, SRM-II). As a precondition for all of these methods, it should be mentioned that the support of the antecedent set has to coincide with the support of the observation. Generally this is not fulfilled. In such cases the fuzzy relation (rule) obtained in the previous step is transformed first, in order to meet this condition.

The **Fuzzy Rule Interpolation based on POlar Cuts** (FRIPOC) The Fuzzy Rule Interpolation based on POlar Cuts [20] solves the task of fuzzy reasoning in two steps conform to the GM. First a new rule is interpolated whose antecedent part is in the same position as the observation in each antecedent dimension. The expression "same position" means that in each partition the reference point of the observation and the reference point of the rule antecedent set are identical. FRIPOC uses the centre of the core as reference point.

The new rule is determined in three stages. First the shapes of the antecedent sets are calculated using the set interpolation technique FEAT-p separately in each antecedent dimension. Its main idea is that all sets of the partition are shifted horizontally into the interpolation point (reference point of the observation), i.e. their reference points will be identical with the interpolation point. Next the shape of the new set is calculated by its polar cuts. For each polar level the polar distance is determined as a weighted average of the corresponding polar distances of the overlapped known sets. The position of the consequent sets is calculated in the second stage using an adapted version of the Shepard interpolation [21]. Next, (stage 3) one calculates the shape of the consequent sets by FEAT-p in an identical way as seen in case of the antecedent sets (stage 1).

The second step of FRIPOC determines the conclusion from the observation and the previously generated auxiliary rule using the method SURE-p. The Single rUle Reasoning based on polar cuts calculates the differences between the polar distances corresponding to the observation and the antecedent of the interpolated rule in each antecedent dimension and for each polar level. Next an average difference is determined for each polar level. One calculates the conclusion by modifying the consequent of the interpolated rule by the average differences followed by a control and correction algorithm in order to ensure the validity of the new fuzzy set.

The method **LEast Squares based Fuzzy Rule Interpolation** (LESFRI) [22]. It also belongs to the group of two-step fuzzy rule interpolation techniques. It uses FEAT-LS as set interpolation technique. FEAT-LS was developed especially for the case when all sets of a partition belong to the same shape type and the characteristic (break) points are also situated at the same α-levels. In such cases it seems to be a natural condition on the new linguistic term created in the interpolation point to suit this regularity as well. As a first step all the sets of the partition are shifted horizontally in order to reach the coincidence between their reference points and the interpolation point. Next, the characteristic points of the new set's shape are determined by the method of weighted least squares taking into consideration the corresponding characteristic points of the overlapped sets. The weighting expresses that the sets situated originally in closer neighborhood of the interpolation point should exercise a higher influence than those situated originally in farther regions of the partition.

LESFRI uses SURE-LS as single rule reasoning method in its second step. SURE-LS applies an α-cut based approach for this task. It uses a set of α-levels compiled together by taking into consideration the break-point levels of all antecedent dimensions and the current consequent partition. The calculations are done separately for the left and right flanks. On each side for each level it calculates the weighted average of the distances between the endpoints of the α-cuts of the rule antecedent and the observation set. The weighting makes possible to take into consideration the different antecedent dimensions (input state variables) with different influence.

The basic idea of the method is the conservation of the weighted average differences measured on the antecedent side. Applying these modifications on the consequent side usually results in a set of characteristic points that do not fit the default set shape type of the partition. Therefore the method of Least Squares is used in order to find the break-points of an acceptable conclusion.

3. Practical Applications

Fuzzy rule interpolation technologies became increasingly important for automatic identification of sparse fuzzy rule-bases from sample data. There are three main application areas of FRI based fuzzy systems: fuzzy control, fuzzy modeling (function approximation applications), and expert systems.

Further on we introduce 3-3 sparse rule-base based applications belonging to the first two groups, and one application representing the third group.

3.1. Fuzzy Control

Kovács and Kóczy [23] applied rule-interpolation based fuzzy reasoning for the simulation of an **automated guided vehicle**. Their main goal was the path tracking and the collision avoidance without losing the designated path. The used approximate fuzzy reasoning method was FIVE.

The obstacle avoidance strategy leaned on three measurements of ultrasonic sensors. The system was simulated on a test path and obstacle-configuration. The applied planning (modification and test) can be very useful for controlling of unknown or partly known systems. The generated fuzzy system had two outputs, the speed and the steering, which was achieved by creating two rule bases. Altogether 12 rules were needed for controlling of steering and 5 for speed. The vague environment of antecedent and consequent universes (scale functions) were generated in a training process, which was based on data collected from human experts. In order to get the shortest docking distance on the test path the training process was optimized for the core positions of the linguistic terms and the values of the scale functions (see in figure 1).

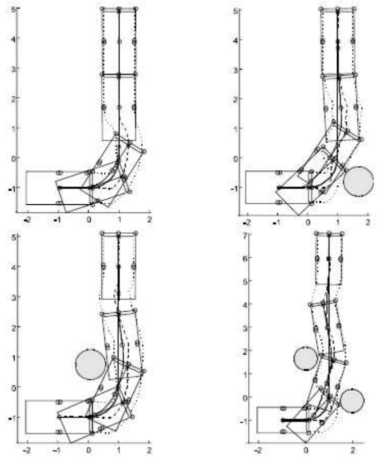

Fig. 1. Some simulated outcomes of AGV [23]

Vincze and Kovács [24] introduced an **automated mobile robot room surveillance navigation control** with FIVE as fuzzy reasoning method. The robot navigated with help of waypoints and it avoided the collisions with the obstacles and walls. If something blocked the way of the robot, it turned around and headed towards the opposite of the last direction. The test configuration had 4 waypoints in fixed order, which were joined to the four corners of the room. The room was oblong and had 4:3 side-ratio (see in figure 2). The navigation control was built from three components, these chose the next waypoint to approach, controlled the avoidance of the walls and the obstacles as well as the changing of the direction.

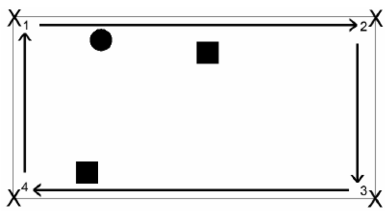

Fig. 2. The room, where the robot (the round object) navigates with the waypoints (in the corners) and the obstacles (the two squares) [24]

The first step of the control was the waypoint selection. The result-vector was added to the actual position of the robot. With this new position was calculated the distance from the walls and the obstacles. Then the rule base of the wall and the obstacle avoidance were evaluated. These results were summed with the actual position and this would be the next valid location of the robot. If it was necessary to modify the moving direction of the robot, the waypoint order was inverted. If one repeats the procedure in loop, gets the model of surveillance navigation controlling and collision avoiding.

If one would build a full covering rule base with the same strategies and 4 waypoints it would need $2^{(2n+2)}$+8+4+4 rules, that are in total 1040 rules. The solution based on a sparse rule base required only n*(6+n)+3+4+4 rules, that is 51 rules. This rule base can be implemented easy even in embedded FRI fuzzy logical controllers in case of high number of input dimensions.

Kovács and Kóczy suggested the application of **an interpolation-based fuzzy reasoning method for behavior-based control structures** in [16]. The solution can be implemented easily and fast enough to fit to the structure of the behavior-based control in real time direct fuzzy logic control systems. In case of pure behavior-based control structures every main task of the control – the behavior control, the behavior fusion and the behaviors ourselves – is implemented in fuzzy controllers.

The main task of the behavior-coordination is to choose the most needed behavior from the known behavior patterns in order to handle the actual situation. The proposed solution estimates the similarities of the preconditions of the actual situation and the known behaviors. This is named symptom evaluation (see figure 3).

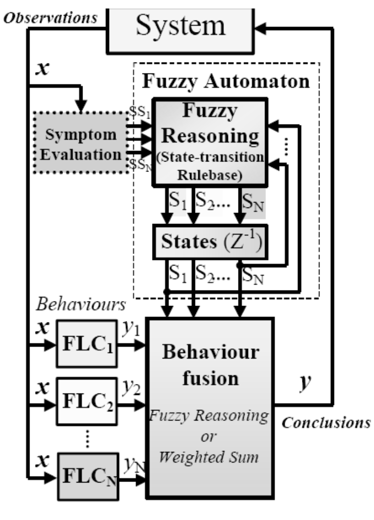

Fig. 3. The proposed behavior-based control structure [16]

The user is handled adaptively with the existing (offline collected) human opinions (user models) combinations. The new state values are given based on the observations (inputs) the conclusion of the user feedback (the evaluation of the symptom in the state transition i into every conceivable state of SS_i $\forall i \in [1, N]$) and the previous S_i state values.

For example if the system has already found a satisfying model (S_i), and the user feedback (SS_i) comes out for it, the system keeps it even if the user feedback starts to come out for other models. The goal of the heuristic is to reach a relative fast convergence, which can be important for numerous application areas, for example for introduction of an online user adaptive choosing system, where the feedback information are limited for the state variables.

If the strategy introduced in [16] would be used for the classical fuzzy reasoning, the covering rule bases would need 16 rules because of two state case (as the observing universe has four dimensions: S_1, SS_1, S_2, SS_2) and each has two fuzzy sets (zero, one). Applying a sparse rule base only 7 rules are needed. The disadvantage of the proposed method is that the result of the reasoning is a singleton fuzzy set. This has not influence in applications where the result is defuzzified.

3.2. Function Approximation Type Applications

Wong and Gedeon [25] reported the generation of fuzzy models for petrophysical properties prediction. One of the key tasks in course of the analysis of petroleum well log data is the prediction of petrophysical properties corresponding to specific input data, i.e. depth values different from the original ones used by the experiments. Such properties are the porosity, permeability and volume of clay [25]. The expensive and time consuming character of the data collection from boreholes increases the significance of the prediction. The predicted values help taking decisions on rentability of the exploration of a specific region. The aim of the research was to establish a low complexity fuzzy model taking into consideration three input variables: the gamma ray, the deep induction resistivity and the sonic travel time. The models have one output parameter. The training sample data set had 71 data rows, and the test data set had 51 data rows. The data were preprocessed, and each variable was normalized to the unit interval.

The applied FRI method was MACI. The prescribed output and the results produced by the model were compared by the correlation factor. The results can be seen in table 1. The generated system was based on 36 rules.

Table 1. Values of correlation factor [25]

Applied method	Correlation factor	
	Training data	Test data
MACI	0.917	0.865

Fuzzy models can adapt to diversified system configurations and operation conditions well. Johanyák, Parthiban and Sekaran [26] introduced models which were prepared based on laboratory experiments for **anaerobic tapered fluidized bed reactor**. The task of the system was the anaerobic digestion of synthetic wastewater derived from the starch processing industries.

The model had four input (Flow rate, Chemical Oxygen Demand [COD], pH, Biological Oxygen Demand [BOD]) and 5 output (COD, Biogas, Volatile Fatty Acids, Alkalinity, BOD) values. The sample data set consisted of 78 data rows. The lower and upper limits of the input and output universes were prescribed values. Four fuzzy models have been prepared using the Automatic fuzzy system generation based on fuzzy Clustering and Projection (ACP) algorithm and the FRIPOC rule interpolation based reasoning method. The models were evaluated using relative value of the root mean square error (RMSEP).

Table 2. The results of the tuning process [26]

	RMSE	RMSEP%
COD	27.7827	4.11
Biogas	0.8012	2.46
VFA	18.2828	7.75
Alkalinity	76.0786	9.67
BOD	88.4201	9.96

Table 2 presents the results of the system evaluations. In case of COD the result is relative good, the deviation was significant only in case of a few measured-computed values. The best result had the system, which modeled the relation between the Biogas and the input. In case of VFA and Alkalinity the results were mediocre. The output parameters predicted by the tuned system were very close to the corresponding experimental results. The model was validated with repeated tests.

The **proper selection of cutting parameters for machining operations** has determinant role in achieving the expected economical and quality goals. Therefore, several models have been developed aiming the reliable prediction of tool life, i.e. the modeling of the functional relationship between the tool life and its main influential factors. Such factors are the cutting speed and the feed rate.

Relevant models are the exponential [27], the Taylor [27], the corrected Taylor [27], the Gilbert [28], and the Kronenberg [29] model. Their parameters can be estimated from experimental tests using some optimization methods but the approximation accuracy decreases when the cutting speed or the feed rate increases. For this task the fuzzy model applying RBE-SI+FRIPOC [30] ensured the best results.

In course of the modeling DA20 and DA25 carbid insert types were examined based on milling experiments. The author developed two separate models for the two carbide insert types. Both models had two input and one output dimensions. They were created using the Sparse Fuzzy Model Identification (SFMI) Matlab ToolBox [31]. The models used RMSEP as a performance index and calculated the performance index with the Fuzzy Rule Interpolation (FRI) Matlab Toolbox [32].

Table 3. The performance indexes (RMSEP) of the models of the tool life [30]

	Exp.	Taylor	T. corr	RBE-SI+ FRIPOC
DA20	1.1223 %	2.8816 %	4.7610 %	0.0146 %
DA25	0.7045 %	3.9486 %	7.2525 %	0.0005 %

Table 3 compares the performance (RMSEP) of the three traditional models and the fuzzy model applying fuzzy rule interpolation

3.2. Fuzzy Expert System

In several cases when a fully automated student scoring is not possible (e.g. narrative responses, software development) the **evaluation of the students' academic performance** can result in quite significant deviation between the marks given by different evaluators or at different occasions. This problem partly can be traced back to the vagueness in the opinion of the evaluator that hardly can be fitted in the one-value-based traditional evaluation model. Student Evaluation based on Fuzzy Rule Interpolation (SEFRI) [33] aims the support of the evaluator by allowing the scoring of each question by fuzzy numbers and by calculating the total score using fuzzy inference.

In course of the rating the evaluator takes into consideration three aspects, namely the accuracy of the response, the time necessary for answering the questions, and the correct use of the technical terms. In course of the preparation the 100 achievable marks are divided between the questions. They are the weights associated to the questions.

In case of the second aspect one works with the total time necessary for answering all of the questions, which is determined automatically and reported to the allowed total response time. The resulting relative time is fuzzified (TR) using singleton type fuzzification.

The characteristics "the accuracy of the response" (AC), and "the correct use of the technical terms" (CU) are measured by the evaluator with separate fuzzy marks (fuzzy numbers) for each question. The scoring scale is in both cases the unit interval. After assigning the two fuzzy marks for each question one calculates an average AC and CU value ($\overline{AC}$ and $\overline{CU}$) for the student as a weighted average of the individual values.

Next one determines from the three fuzzy values ($\overline{AC}$, TR, and $\overline{CU}$) the general evaluation of the student using fuzzy inference. In order to reduce the complexity of the rule base the LESFRI fuzzy rule interpolation based reasoning method is used. Thus the underlying rule base requires only 64 rules in contrast with the 125 rules of the dense rule base owing to the fact that each input dimension contains five fuzzy sets.

The fuzzy inference results the general fuzzy evaluation of the student (GFE) that is defuzzified using Center Of Area method in order to get the total score (TS). Finally the grade of the student is determined using the standardized mapping of the university.

4. Conclusions

Fuzzy rule interpolation based inference is an emerging field of fuzzy set theory allowing the reasoning even in cases when not all the possible rules are known or the rule base is intentionally sparse in order to ensure a low system complexity. A wide number of practical applications demonstrate that FRI is not only of theoretical interest. Although originally it was developed for fuzzy systems with high number of input dimensions and partitions with high resolution it also can be used successfully in other cases. The known practical applications form three main groups conform the tasks they are used for: fuzzy control, function approximation and expert systems.

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