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FUZZY SET APPROXIMATION BASED ON LINGUISTIC TERM SHIFTING

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Fuzzy systems working with a sparse rule base apply special reasoning techniques in order to ensure the acceptable output in case of observations hitting the gap between the antecedent parts of the rules, too. The methods that follow the generalized methodology of the fuzzy rule interpolation (GM) [1] produce the conclusion in two steps. First a new rule is interpolated in the position determined by the reference point(s) of the observation followed by the estimation of the conclusion by firing this rule. The new fuzzy set approximation method (FEAT-α) proposed in this paper offers a solution for the task of the first step of the GM. Its main features are its low computational complexity, its ability to take into consideration all the sets belonging to the partition and the fact that the calculations are based on α-cuts.

1 INTRODUCTION

Systems working with fuzzy logic produce their output using a rule-based inference technique. One of the more important features of their knowledge base is the sparse or dense character of the rule base. The collection of the rules can be considered sparse when there exist possible observations (input values) which do not overlap the antecedent part of any rules.

The classical fuzzy reasoning techniques like Zadeh's, Mamdani's, Yager's or even Sugeno's cannot afford an acceptable output in such cases. Therefore such fuzzy inference methods are applied in systems built on sparse rule bases, which can estimate the result in lack of proper rules, too.

These techniques can be divided into two groups depending on whether they are producing the estimated conclusion directly or they are interpolating an intermediate rule first. The structure of the methods belonging to the second group can be described best by the GM defined by Baranyi et al. in [1].

A Fuzzy sEt Approximation Technique (FEAT) based on the shifting of the linguistic terms and its α-cut based implementation (FEAT-α) is suggested in this paper. This method was developed for the first step of the generalized methodology.

The rest of this paper is organized as follows. First the concept of the GM is recalled, followed by the presentation of the proposed FEAT method and some numerical examples outlining the sensitivity of the technique.

2 GENERALIZED METHODOLOGY OF THE FUZZY RULE INTERPOLATION

The generalized methodology (GM) of the fuzzy rule interpolation was introduced by Baranyi et al. in [1]. It defines a reference point (RP) for the characterization of the position of the fuzzy sets. There are several possible choices regarding to it, e.g. the centre of the core [1] [2], the centre of the support [2], etc. can play this role. It can be considered as a parameter of the technique. The distance of the fuzzy sets, which plays a crucial role in the determination of the approximated result is measured by the Euclidean distance of the reference points of the sets (1).

$$d(A_1, A_2) = |RP(A_1) - RP(A_2)| \tag{1}$$

where A_1 and A_2 are the fuzzy sets, RP is the reference point and d is the distance of the sets.

The GM consists of two steps. First a new rule is determined whose antecedent part overlaps the observation at least partially and beside this in the case of a one dimensional antecedent universe the RP of the antecedent set is the same as the RP of the observation. In case of a multiple dimensional antecedent universe the last statement is valid for all the dimensions.

The approach of the GM reflects the assumption that there exists regularity in the rule base expressing a continuous mapping between the antecedent and consequent universes. In the first step the concrete form of this mapping is determined corresponding to the reference point(s) of the observation. The approximation process of the new rule can be divided into three stages. First the antecedent is determined using a set interpolation/approximation technique. Secondly the position of the consequent is calculated using for example the fundamental equation of the fuzzy rule interpolation (FEFRI) [5][1]. Thirdly the shape of the consequent is determined applying the same technique and approach as in the first stage.

In the second step of the GM the approximated conclusion is determined by firing the new rule. Usually the antecedent part of the rule and the observation does not coincide perfectly, therefore a special single rule reasoning technique is needed.

3 FUZZY SET APPROXIMATION BY SHIFTING OF LINGUISTIC TERMS (FEAT-α)

The proposed method is based on the assumption that a better approximation of the real relation between the antecedent and consequent universes can be attained by taking into consideration all the available rules in the rule base. This supposal already appeared right at the beginning of the history of the fuzzy rule interpolation e.g. in [5] or later e.g. in [7]. In spite of the possible advantages most of the methods use only two rules that surround the observation. The technique being presented serves the determination of the antecedent and consequent sets of the new rule in the first and third stage in the first step of the GM. The method is the same regardless of being applied for an antecedent or a consequent dimension.

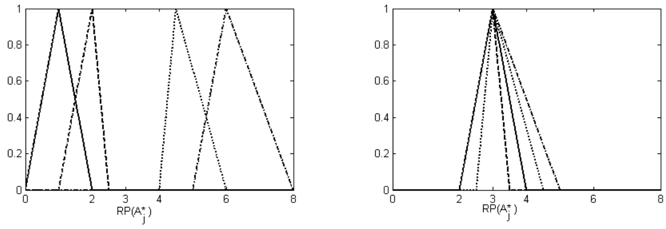


Figure 1. The original partition and the result of the shifting

The starting point is a fuzzy partition with the reference points of the sets determined in advance and the reference point of the observation in the actual dimension/partition. All the sets in the partition belong to the antecedent part of one or more rules.

First all the sets are shifted horizontally in order too reach the coincidence of their reference points with the reference point of the observation. This idea is similar to the concept in [2], but that method uses and translates only the two flanking sets into the location of the observation.

Next the shape of the new set is determined from the collection of the overlapped sets. There are several solutions for this task. Further on an α-cut based technique (FEAT-α) with low computational complexity is proposed for the most popular case of the convex and normal fuzzy (CNF) sets. In [3] the authors introduce the concept of the polar cut and based on it they suggest a solution (FEAT-p) for the case when the normality condition is not satisfied for all the sets participating in the approximation process, i.e. the height of one or more sets is smaller than 1. Similar to the choice of the reference point the selection of the calculation mode of the shape is also a tuning point of the technique.

The suggested shape calculation technique is based on the extension and resolution principle of the fuzzy sets. For each α-cut the raw (starting) values for the lower and upper endpoints are determined as weighted averages of the endpoints of the α-cuts of the shifted sets using the formulas (2) and (3).

$$\inf\left\{A^{a}_{j\alpha}\right\} = \frac{\sum_{k=1}^{n_j} w_{jk} \cdot \inf\left\{A_{jk\alpha}\right\}}{\sum_{k=1}^{n_j} w_{jk}} \tag{2}$$

$$\sup\left\{A^{a}_{j\alpha}\right\} = \frac{\sum_{k=1}^{n_j} w_{jk} \cdot \sup\left\{A_{jk\alpha}\right\}}{\sum_{k=1}^{n_j} w_{jk}} \tag{3}$$

where *inf* and *sup* denote the lower and upper endpoints of an α-cut, j is the actual antecedent dimension, α is the level of the actual cut, n_j is the number of the sets in the partition, $A_{jk\alpha}$ is the α-cut of the k$^{\text{th}}$ set, w_{jk} is the weighting factor of the k^{th} set and $A^{a}_{j\alpha}$ is the approximated α-cut.

It seems to be natural that the sets whose original position were in the neighbourhood of the reference point of the observation to exercise higher influence as those ones situated in farther regions of the universe of discourse. Therefore the weighting factor should be dependent on distance. The simplest weighting factor is the reciprocal value of the distance, but there are several recommendations in the literature for more or less analogue cases. For example in [5] the square of the reciprocal value of the distance is suggested (p=2, λ=1) (4). The authors of [6] propose the use of the reciprocal value of the distance on the m^{th} power (p=m, λ=1), where m is number of the antecedent dimensions. In [7] three variants of the weighting factor called extensibility functions are introduced. These can be described by (4) with p=1 respective p=2 and (5).

$$w_{jk} = \frac{1}{\lambda \cdot d\left(A_j^*, A_{jk}\right)^p} \tag{4}$$

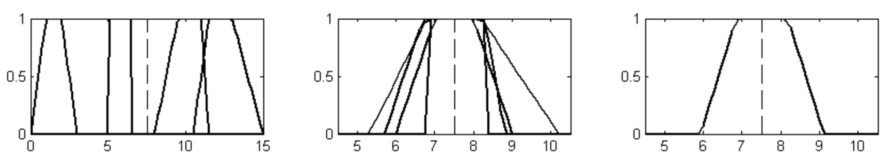
$$w_{jk} = e^{-\lambda \cdot d\left(A_j^*, A_{jk}\right)} \tag{5}$$

where A_j^* is the fuzzy set corresponding to the j^{th} dimension of the observation, λ is a positive constant determining the effective extensibility distance. The choice of the weighting factor can add a free parameter to the formula (2) and (3) to adjust the sensitivity.
If the method FEAT-α starts with a partition containing only convex fuzzy sets the convexity feature of the approximated set is also expectable. Therefore the raw shape goes through a verification and correction stage (VCS). The VCS starts with the highest α-level and advances in top-down direction using the formula (6) for the verification. If it is necessary the α-cut is enlarged to ensure the fulfilment of the condition (6).

$$\inf\left\{A^a_{\alpha i+1}\right\} \le \inf\left\{A^a_{\alpha i}\right\} \le \sup\left\{A^a_{\alpha i}\right\} \le \sup\left\{A^a_{\alpha i+1}\right\} \qquad \alpha_{i+1} < \alpha_i \tag{6}$$

4 NUMERICAL EXAMPLES

This section intends to present some relevant features of the suggested method outlining the effect of the weighting factor.



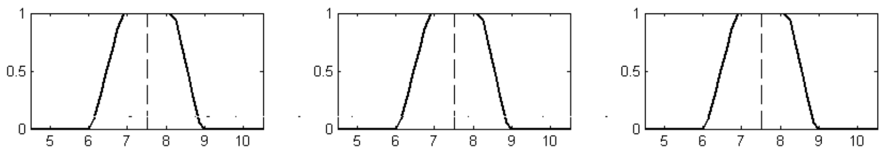
a. The original partition   b. Shifted sets   c. w=1

Figure 2.

Figure 2.a shows a partition containing four trapezoidal shaped fuzzy sets with the characteristic points given by (7) and (8). The shifted sets corresponding to the reference point of the observation RP(A*)=7.5 are presented in figure 2.b. Figure 2.c contains the approximated set using the simplest weighting factor (w_i=1).

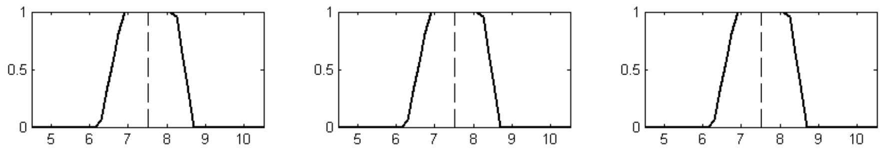
$$A_1 = \begin{bmatrix} 0.0 & 0 \\ 1.0 & 1 \\ 2.0 & 1 \\ 3.0 & 0 \end{bmatrix}, \qquad A_2 = \begin{bmatrix} 5.0 & 0 \\ 5.0 & 1 \\ 6.5 & 1 \\ 6.5 & 0 \end{bmatrix} \tag{7}$$

$$A_3 = \begin{bmatrix} 8.0 & 0 \\ 9.5 & 1 \\ 11.0 & 1 \\ 11.5 & 0 \end{bmatrix}, \qquad A_4 = \begin{bmatrix} 10.5 & 0 \\ 11.5 & 1 \\ 13.0 & 1 \\ 15.5 & 0 \end{bmatrix} \tag{8}$$



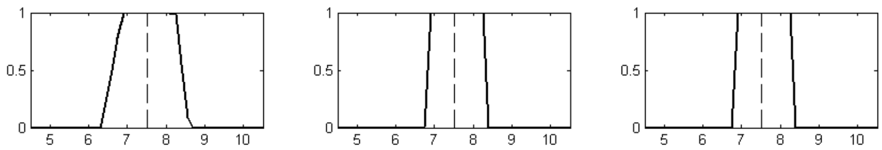
a. λ=1 b. λ=10 c. λ=100

Figure 3.



a. λ=1 b. λ=10 c. λ=100

Figure 4.



a. λ=1 b. λ=10 c. λ=100

Figure 5.

The results presented in figure 3 were obtained using the weighting factor (4) with p=1. The results presented in figure 4 were obtained using the same weighting factor with p=2. Figure 5 contains the approximated sets by weighting factor (5). In each case (Fig. 3-5) three λ values were tried. It can be clearly observed that increasing λ the second set (A_2), which is the nearest one to the observation becomes more and more dominant. The weighting factor (5) is the most sensitive to the value of λ.

5 CONCLUSIONS

Fuzzy systems built on sparse rule bases use inference techniques based on rule interpolation in order to ensure the acceptable output even in the cases when there are no rules whose antecedent part would overlap the observation at least partially. A group of these techniques follow the generalized methodology of the fuzzy rule interpolation [1].

In this paper an α-cut based new method is proposed for the first step of the GM. Its advantages are its low computational complexity, comprehensibility and its assumed positive influence on the approximation capability of the GM. The basic idea is similar to the fuzzy rule interpolation method Interpolative Reasoning Based on Graduality introduced in [2]. Contrary to the IRBG the method FEAT-α aims a set approximation, it uses all the sets belonging to the partition and the shape of the linguistic term is calculated in a different mode.

Some numerical examples were presented in order to outline the sensitivity of the technique to the selection of the type and parameters of the weighting factors.

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